Using multipole solutions to compute linearised steady soil water flow around systems of obstacles

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A nonlinear Richards' equation governs the steady unsaturated flow of water through soil, and the Gardner approximation transforms this to a linear equation for the soil water potential. The Gardner equation has the form of an advection diffusion equation with uniform advection in the vertical direction, which can be transformed to a Yukawa or modified Helmholtz equation with well known multipole solutions. An impermeable obstacle in the soil has a no flow boundary condition on its surface. For a spherical or cylindrical obstacle this becomes a radiation type boundary condition for the Helmholtz equation, with the coefficient depending on angle. For a single spherical or cylindrical obstacle in steady downward flow there is an explicit solution as an infinite series of multipoles with known coefficients.

Using an addition theorem for multipoles this solution has been extended to the flow around an obstacle near a source or line solution. Using addition theorems the problem of soil water flow around several obstacles or an infinite array of obstacles can be written as the solution of an infinite set of linear equations for the coefficients of a multipole series corresponding to each obstacle. These sets of equations are readily truncated and solved. The streamlines and isochrones are calculated by numerical integration of the series solution for the velocity field.