

**Inverse Problems: A Pragmatist's Approach to
the Recovery of Information from Indirect Measurements**

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ABSTRACT

Within the class of inverse problems, it is the subclass of indirect measurement problems that characterize the nature of inverse problems that arise in applications. With very few exceptions, measurements only record some indirect aspect of the phenomenon of interest (e.g. X-rays and tomographic images in medical applications; telescope images in astronomy; stereological assessment of biological structure and processes; signatures in geophysical prospecting). Even when the direct information is measured such as weight or length, it is measured as a correlation against a standard and this correlation can be quite indirect, such as the measurement of weight by the extension (compression) of a spring.

The recovery of information about a phenomenon from indirect measurements is a piecemeal process. Any class of indirect measurements can only recover certain information about the phenomenon. In order to formulate realistic mathematical models that relate the indirect measurements to the specific information of the phenomenon that is to be recovered, there is a need to invoke simplifying assumptions (e.g. radial or axial symmetry). The required information about the phenomenon is often only vaguely contained in the available indirect measurements.

All these aspects influence how the recovery of the information can be performed. The choice of methodology is not limited. The challenge is to perform the recovery in a way that correctly reflects the underlying nature of the problem context. It is not a matter of blindly applying some form of quadratic regularization for which algorithms and packages are readily available. Though such tools are useful for initial exploratory analysis, the crucial characterization of the information to be recovered is hidden in the mathematical model that relates the indirect measurements to the phenomenon within the problem context.

When recovering information from indirect measurements, the question that focuses the problem-solving comes from the need for decision-making to have answers to specific matters. The data available for the associated decision-support will be indirect measurements of the phenomenon under investigation. As a consequence of the

applications context, the recovery of information of the associated inverse problem will be constrained by practical challenges including:

- (i) In a given situation, how does one decide on the indirect measurements to be performed?
- (ii) How are some practical people able to solve indirect measurement problems without having to perform an explicit regularization?
- (iii) Is there any advantage in combining different indirect measurements of the same phenomenon?
- (iv) What are the alternatives, when there is only a (very) limited amount of data?
- (v) How does one proceed when a mathematical model is not available or is too complex to formulate?

The talk will examine, in terms of practical problems, how such challenges can be accommodated.

Bob Anderssen grew up in the country in Queensland. He completed an MSc in applied and computational mathematics at the University of Queensland. His PhD in mathematics is from Adelaide. He taught mathematics for one year at Monash before accepting a full time research position in the ANU in computational mathematics. In 1979, because of his keen interest in the application of mathematics to real-world problems, he accepted a position in industrial and computational mathematics in the CSIRO. He has held visiting positions at a number of international universities including Stanford, Princeton, Cambridge (UK), TU-Munich and TU-Vienna, and given invited lectures at an even bigger group including Harvard, Oxford, Ecole Polytechnique and Oberwolfach. He has been president of the Australian Mathematical Society and chair of the National Committee for Mathematics. His current research has focused on theoretical polymer dynamics, vibrating piano strings (the Stuart piano), the flow and deformation of wheat flour dough from a plant breeding perspective and the drying of pasta. His hobbies include gardening, hiking and classical music.