A Hybrid Quasi Monte Carlo Method for Estimating the Reliability of Multistate-node Acyclic Networks

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ABSTRACT— A system where components and system itself are allowed to have a number of performance levels is named Multi-state System (MSS). A multistate-node acyclic network (MNAN) is a generalization of the tree-structured MSS without satisfying the flow conservation law. Evaluating the MNAN reliability arises at the design and exploitation stage of many types of technical systems. Up to now, the known existing methods are still impossible to evaluate the MNAN reliability in a reasonable time even for a smaller sized problem. Hence, the main purpose of this article is to present a simple Hybrid Quasi Monte Carlo method for estimating the reliability of a MNAN to improve calculation efficiency. One example is illustrated to show how to evaluate MNAN reliabilities by the proposed Hybrid Quasi Monte Carlo simulation algorithm.

Key Words – Network Reliability, Multi-state System (MSS), Multistate-node Acyclic Network (MNAN), Hybrid Quasi Monte Carlo Method, Minimal Tree/Cut

1. INTRODUCTION

In recent years, network reliability theory has been applied extensively in many real-world systems such as computer and communication systems, power transmission and distribution systems, transportation systems and etc [1-9]. Thus, the system reliability plays important roles in our modern society. It is recommended to be measured and evaluated the performances of the systems which can be modeled as stochastic networks or into fault trees first.

Traditional binary-state reliability theory is dealing only with system and elements that have two possible states - complete failure and perfect functioning. Modern large-scale technical systems are distinguished by their structural complexity. MNAN is a special MSS. In MNAN, each node has different states determined by a set of nodes that receive the signal directly without satisfying the conservation law [10-15]. The system has a source node that can only emit and send a signal to other nodes, a number of sink nodes that can only receive a signal, and a number of intermediate nodes (neither source nor sink nodes) that retransmit the received signal to some other non-source nodes. The signal is transmitted from a non-sink node to a number of non-source nodes along edges between these nodes, i.e. multistates. No signal leaving a node can return to this node through any sequence of nodes. The probability of which nodes (states) are transmitted to next is assumed to be known for each non-sink node. All of these probabilities are assumed to be statistically independent. MNAN is more practical and reasonable than traditional network (TN) which satisfies the conservation law in many real-life situations such as computer networks, cellular telephone networks [8-17], etc. Therefore, MNAN analysis has become a new subject in system reliability.

MNAN was first investigated by Malinowski and Preuss [10]. Multi-state linear consecutively-connected networks were introduced by Hwang & Yao [11] and studied by Kossow & Preuss [12] and Zuo & Liang [13]. An algorithm based only on the universal generating function technique (an implicit enumeration procedure) to find all of the reliabilities in terms of MTs between the source node and the subset of sink nodes was first proposed by Levitin [14]. Yeh improved Levitin's algorithm using a special Branch-and-Bound algorithm (an efficient implicit enumeration procedure) [15] further. Yeh proved that the traditional binary-state networks reliability algorithm for TN can easily be revised to solve the MNAN reliability problem [16]. Yeh also proposed the best-known algorithm for the MNAN reliability problem in terms of the minimal cuts (MCs) [17]. These existing known algorithms [14-17] are all still impossible to evaluate the MNAN reliability in a reasonable time even for a smaller sized problem. The need for an efficient and intuitive method to evaluate the MNAN reliability of thus arises.

Generally in MNAN reliability evaluation, all MTs/MCs of the network must be known in advance [6-17]. However, both the problems in finding all MTs/MCs and computing the system reliability in terms of the known MTs/MCs are NP-hard [14-17]. To overcome both NP-hard obstacles completely and reduce the computational burdens, the simulation method is introduced here to estimate the MNAN reliability. As far as the author is aware, none of the simulations have been proposed in the literature to evaluate the

MNAN reliability. The Quasi Monte Carlo simulation is the traditional Monte Carlo simulation but using the quasi-random sequences instead (pseudo) random numbers. In several cases, it permits to improve the performance of Monte Carlo simulations, offering shorter computational times and/or higher accuracy [18,19].

The purpose of this study is to develop a simple Hybrid Quasi Monte Carlo simulation algorithm combining pseudo-random numbers and quasi-random numbers to evaluate the MNAN reliability without needing to search for MTs/MCs in advance.

2. ACRONYM, NOTATION AND NOMENCLATURE

Acronym

MSS	Multi-state System
MNAN	Multistate-node acyclic network
MT	Minimal Tree
MC	Minimal Cut

Notation

$\|\bullet\|$	the number of elements in •.
$E[\bullet]$	the expected value of •.
$Var[\bullet]$	the variance of •.
$P\{\bullet\}$	the probability of event •.
$G(V, E)$	a MNAN with the set of nodes $V=\{1,2,\ldots, n\}$ and the set of edges E, where node 1 is the source node. The nodes are numbered in such a way that for any edge $e_{uv}\in E$ with $u<v$ [14].
T	the target node set ($\subseteq V-\{1\}$) of a MNAN.
s_i	the number of states of node i, where $i=1,2,\ldots,\|V-T\|$.
S_{ij}	the jth state of node i, where $i=1,2,\ldots,\|V-T\|$ and $j=1,2,\ldots, s_i$.
$p_{ij}(t)$	the probability of all nodes in S_{ij} receive a signal directly from node i at time t, where $\sum_j p_{ij}(t)=1$ for $j=1, 2, \ldots, s_i$.
R_{iJ}	the probability of all nodes in J ($\subseteq V-\{1\}$) receiving a signal directly from node i.

m	the total number of independent trials.
$R(t)$	the MNAN reliability at time t.
$\rho(t)$	the steady-state reliability estimator of $R(t)$.
$R_i(t)$	$\rho(t)$ of the ith simulation trial for i=1, 2,…, m.
$X_i(t)$	the state of node i at time t for i= 1, 2,…, n. In fact, $\{X(t), t\geq 0\}$ is a stochastic process.
$\phi(t)$	the structure function of the system at time t. In fact, $\phi(t)=1$, if the system under state $X_1(t), X_2(t),\ldots,X_n(t)$ is operative and $\phi(t)=0$, otherwise.

Nomenclature

Acyclic network: a network containing no directed cycle.

Target set/node: a non-empty subset of sink node set and its element is called the target node.

States: it is a subset of nodes that one node can transmit signal to.

MNAN Reliability: The reliability of a MNAN at time t, $R(t)$, is defined as the probability that a signal can be transmitted from the source node to the target set which operates successfully throughout the time interval (0, t]. It can be denoted by $R(t)=P\{$there exists at least one path from node 1 to any target node in $G(V, E)$ during $(0, t)\}=P\{\phi(t)=1\}$.

the flow conservation law: the total flow (signal) through into and from a node (not a source node, target nodes, or a target node) are all equal.

3. THE PROPOSED QUASI MONTE CARLO METHOD

The quasi-random sequence used here is adapted from the van der Corput sequence. Consider the radical inverse function as follows:

$$\Phi_b(k)=\sum_{j=0}^{i} a_j(k)b^{-j-1} \text{ when } k=\sum_{j=0}^{i} a_j(k)b^{j}\ , \qquad (1)$$

where k is the corresponding van der Corput sequence, $a_j(k)$ is the digits sequences, the natural number b>1 is the base, and i is the lowest positive integer that makes $a_j(k)=0$ for all $j>i$. The value $\Phi_b(k)$ always are in the unit interval [0,1) [16]. The complete proposed Hybrid Quasi Monte Carlo method which estimates MNAN reliability is given as follows:

0: Let i=1, S={1}, and $R_i(t)$=0 for i=1, 2, …, m.

1: Choose the smallest number, say k, in S.

2: Call Hybrid-Quasi-Random() to generate a random number Φ between 0 and 1.

3: Find the smallest integer, say c, s.t. $\Phi \le \sum_{j=1}^{c} p_{kj}$, where c=1,2,…, s_k.

4: If $T \cap S_{kc} = \varnothing$, then $S \leftarrow S \cup S_{kc} - \{k\}$. Otherwise, go to STEP 6.

5: If $S=\varnothing$, then go to STEP 7.

6: $R_i(t) \leftarrow R_i(t)+1$.

7: If $i<m$, then $i \leftarrow i+1$ and go to STEP 1. Otherwise, go to STEP 8.

8: $\rho(t)=\dfrac{\sum_{i=1}^{m} R_i(t)}{m}$.

Procedure Hybrid-Quasi-Random()

0: Let b=2, k be a positive integer random, Φ=0 and B=1.

1: Let $B \leftarrow \dfrac{B}{b}$, $q \leftarrow$the quotient of $\dfrac{k}{b}$, $r \leftarrow$the reminder of $\dfrac{k}{b}$, $\Phi \leftarrow \Phi + rB$, and $k \leftarrow q$**.**

2: If k>0, then go to STEP 1**.** Otherwise, return Φ.

Even for a binary-state network with n nodes, it has 2^n possible system states. Therefore, m trials in the Hybrid Quasi Monte Carlo method can account at most for only a small portion of states even when n is moderate in size of the MNAN. This limitation introduces sampling error. Hence, the statistical methods are utilized here to analyze output during the simulation activities by the estimator of $R(t)$.

The fact that the Monte Carlo method provides an unbiased estimator of the reliability of a system is well known to the technical community. Hence, we have the following statements of which proofs can be found in any basic probability or simulation book:

Theorem 1. Let $\rho(t)=\dfrac{\sum_{i=1}^{m} R_i(t)}{m}$, then $\rho(t)$ is an unbiased and consistent estimator of $R(t)$ with variance $R(t)[1-R(t)]/m$, where $R(t)$ is the exact reliability and m is the trial number.

If the precision relative error ε and the confidence interval $(1-\alpha)\%$ are given for the n components. Then total number of trials (m) needed in the proposed method can be determined by the following theorem.

Theorem 2. If the relative error ε and the confidence interval $(1-\alpha)\%$ for the simulation are required, then the total number of trials of the simulation must be taken at least $m \geq \frac{Z_\alpha^2}{4\varepsilon^2}$, where $Z_\alpha = \frac{\rho(t) - R(t)}{\sqrt{Var[\rho(t)]}}$.

4. CONCLUSIONS

In this paper the Hybrid Quasi Monte Carlo simulation of the reliability is considered. The suggested Hybrid Quasi Monte Carlo algorithm for the MNAN reliability simulation is an intuitive and simple method combining the pseudo-random numbers and quasi-random numbers. All MTs are unnecessary to be known in advance. It yields to very good results compared to the traditional Monte Carlo by means of the pseudo random numbers.

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