Measuring, pricing and hedging financial risk in a dynamic world

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A large number of strategies are proposed on financial markets to control risks induced by market fluctuations. Traditional or more sophisticated (exotic) financial products are used by businesses or investors to transfer their risks to financial institutions.

Important questions faced by the market risk industry, e.g the development of asset pricing models and hedging strategies based on daily (infinitesimal) risk management criteria, have found answers through probabilistic tools and concepts, such as Brownian motion, martingales, and stochastic control. This in turn has had an important impact on the - exponential - development of this industry.

Numerical implementation is the cornerstone of the modelling process. The choice of a model is hence driven not only by its theoretical properties but also, and in great part, by its numerical tractability. This includes, among other things, reliable estimation of parameters, which are inferred from observable market data (financial products prices). Over the last few years, techniques for stabilizing these ill-posed problems via fast and accurate algorithms in partial differential equations have been developed.

In other respects, with motivation coming from multidimensional problems, Monte Carlo methods have been revisited in order to obtain accurate numerical results for prices of high-dimensional products or their derivatives with respect to key parameters. Efficient methods are based on differentiation on Wiener space and Malliavin calculus. A recent topic is that of solving optimization problems (optimal stopping times, optimal portfolio) through Monte Carlo methods.

Thanks in part to the size reached by derivatives markets, market authorities now require financial institutions to compute their daily global exposure (Value at Risk) via their own "internal" models. Motivated by this challenge, academic and risk-managers are debating the "best concept" of risk measure and various problems induced by the high-dimensional character of the covariance matrices they deal with.

Mathematical finance is a challenging and fast evolving domain, constantly looking for new ideas and concepts in Mathematics. One of its very remarkable aspects is the ability of theoretical research to have direct implications on daily market practice.