On the existence of non-constant volatility in the Bachelier and Black-Scholes formulae

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This paper looks at the existence of non-constant volatilities that agree with the Bachelier and Black-Scholes formulae for all strikes. More specifically, we ask the question of whether one can find a volatility process θ_t such that

$$\forall K, \qquad \mathbf{E}[(S_T - K)^+ | \mathcal{F}_t] = C(T, t, K, \theta_t, S_t),$$

where $C(T, t, K, \sigma, z)$ denotes the expressions for the call options for the Bachelier and Black-Scholes models respectively.