An Examination of the Effect of Non-Normality on Optimal Portfolio Construction: A Copula Based Approach

Anthony Hatherley

UQ Business School, The University of Queensland, Brisbane 4072, Australia
[a.hatherley@business.uq.edu.au]

Tactical asset allocation decisions that are dependent on Markowitz's Mean Variance Portfolio Theory (MVPT) rely heavily on assumptions of normality. Asset returns are assumed to be normally distributed, whilst the dependence structure between assets follows a multivariate normal distribution.

A well known empirical result is that the returns of many asset classes are non-Gaussian [1]. This raises concerns over the appropriateness of the use of MVPT for the purpose of portfolio selection. In particular, MVPT is used to determine the optimal portfolio choice given either the expected return or standard deviation of returns [2]. Furthermore, the dependence structure between assets, given by the correlation matrix, is multivariate normal. The choice of portfolio under this setting does not take into account other possible characteristics of the return distribution (such as skewness or kurtosis) or alternative relationships between assets (such as tail dependence) [3].

The aim of this project is to determine the significance of non-Gaussian assumptions on portfolio selection and optimization across a small, but representative set of asset indices in the context of Tactical Asset Allocation. In particular if non-normal assumptions produce a more efficient frontier, how inefficient are portfolios based on normally distributed returns? What additional risk do fund managers incur by assuming normally distributed returns and dependence structures?

This investigation relies on the application of a class of mathematical functions, known as copula functions, in describing the dependence structure between assets. Copula functions allow for a joint distribution to be constructed using only the observed marginal distributions of the individual assets, and can incorporate a broad range of non-Gaussian dependency structures. In the context of this investigation, it follows that the user is not constrained in their choice of distribution function for asset returns.

Therefore, non-Gaussian distribution functions may be specified for the return distribution of individual assets (these distribution functions need not be the same). Next, the choice of copula can be used to represent different characteristics between assets that are typically not captured by the covariance matrix (such as fat tails or tail dependence). The resultant efficient frontiers produced under non-normality (with possible concordance interrelations) can then be statistically compared to frontiers generated under MVPT.

1. Ang, A & Chen, C. (2002). Asymmetric correlations of equity portfolios. *Journal of Financial Economics*. **63**, 443–494.
2. Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*. **7**(1), 77–91.
3. Rasmussen, M. (2003). *Quantitative Portfolio Optimisation, Asset Allocation and Risk Management*. Palgrave Macmillan, London.