# SMOCS-05

# Program and Abstracts

**Conference on Stochastic Modelling of Complex Systems**

Supported and organized by the
*ARC Centre of Excellence for Mathematics and Statistics of Complex Systems,*
and also sponsored by the
*Australian Mathematical Sciences Institute* and the
*ARC Financial Integrity Research Network*

Daydream Island, Queensland, Australia

10-16 July, 2005

# On the distributions of maximum downfalls of a Brownian motion with drift

Albert N. Shiryaev

*Steklov Institute of Mathematics, 8 Gubkina st., Moscow 119991, Russia*
[albertsh@mi.ras.ru]

Let $B = (B_t)_{t\geq 0}$ be a Brownian motion with drift ($B_t = \mu t + W_t$, where $W = (W_t)_{t\geq 0}$ is a standard Wiener process). We consider the problem of finding distributions of the following characteristics which show how big are drops "from a peak to a bottom" of the trajectories of $B$ on time interval $[0, T]$:

$$D_T = \max_{0\leq s\leq s'\leq T}(B_s - B_{s'})$$

(the largest drop from a peak to a bottom),

$$\overline{D}_T = B_{\sigma_T} - \min_{\sigma_T\leq s'\leq T} B_{s'}$$

(the largest drop from the absolute maximum $B_{\sigma_T}$, where $\sigma_T = \min\{0 \leq s \leq T : B_s = \max_{0\leq u\leq T} B_u\}$ to the (partial) minimum on interval $(\sigma_T, T]$),

$$\underline{D}_T = \max_{0\leq s'\leq \sigma'_T} B_{s'} - B_{\sigma'_T}$$

(the largest drop from the (partial) maximum on interval $[0, \sigma'_T)$ to the absolute minimum $B_{\sigma'_T}$, where $\sigma'_T = \min\{0 \leq s \leq T : B_s = \min_{0\leq u\leq T} B_u\}$).

We prove, in particular, that

(a) $D_T = \max(\overline{D}_T, \underline{D}_T)$, $\overline{D}_T \overset{\text{law}}{=} \underline{D}_T$;

(b) $D_T \overset{\text{law}}{=} \max_{0\leq s'\leq T} |X_{s'}|$;

(c) $\overline{D}_T \overset{\text{law}}{=} \max_{g_T\leq s'\leq T} |X_{s'}|$,

where $X = (X_t)_{t\geq 0}$ is a "bang-bang" process,

$$dX_t = -\mu \operatorname{sgn} X_t\, dt + dW_t, \qquad X_0 = 0,$$

and $g_T$ is the last zero of $B$ before time $T$.

We present also results about explicit formulae for distributions of $D_T$ and $\overline{D}_T$ for the case $\mu = 0$ and give a double Laplace transform for the case $\mu \neq 0$.

# Noncausal Problems in Stochastic Calculus

Shigeyoshi Ogawa

*Department of Mathematics, Ritsumeikan University, Kusatsu-shi, Shiga 525-8577, Japan* [ogawa-s@se.ritsumei.ac.jp]

The stochastic calculus is a calculus with respect to an underlying basic stochastic process, like the Brownian motion, say $Z_t$ $t \in I$. It concerns the diferentiation and integration with respect to the $Z_t$ of such random functions that appear as functionals of the $\{Z_t,\ t \in I\}$. The stochastic calculus originated by K.Itô in 1942 is founded on the fundamental *Hypothesis of Causality*, saying that; every random function $f(t, \omega)$ should be adapted to the increasing familly of $\sigma$-fields $\mathcal{F}_t$ generated by the $Z_t$. The hypothesis seems well fit to the principle of causality in physical sciences, where the variable "$t$" appears as time parameter. Moreover it endowes the theory a remarkable situation of being in natural concordance with the notion of *martingale* which plays indeed an essential role in Itô's Calculus.

Nevertheless the hypothesis of *Causality* gives a disagreable shade on the applicability of the causal theory of stochastic calculus. This can be seen immediately, for example when we think of the case that "$t$" stands for the *space* parameter, or in such case where the parameter "$t$" is multi-dimensional (that is, "a stochastic calculus" for the random field, [2]). The notion of Causality looses its sound meaning in such cases because of the lack of natural sense of *time direction.* Even in the case of physical problems where "$t$" appears as *time parameter*, we can find various situations of noncausal nature, such as the Cauchy problem in the theory of Brownian particle equations [3], noncausal version of the Black-Sholes model in Mathematical Finance [4],the White noise analysis [1] etc. These were the motivations for the author to introduce the noncausal theory of stochastic calculus in 1979, based on the noncausal stochastic integral which is often refered by author's name.

In this talk we will give a unified sketch of the noncausal theory of stochastic calculus as well as of its recent development. We will also refer to some typical applications of the theory to mathematical sciences.

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# Computing convergence rates for denumerable Markov chains

Flora Spieksma

*Department of Mathematics, University of Leiden, Niels Bohrweg 1, 2333CA Leiden, Holland* [spieksma@math.leidenuniv.nl]

Consider an irreducible, aperiodic Markov chain $\{X_n\}_{n=1,2,\ldots}$ in discrete time, on a denumerable state space $\mathcal{S}$. Suppose that there exists a contractive Lyapunov function $f$ for this chain. That is, there exist a positive function $f$ on $\boldsymbol{S}$, an *exception set* $A$, with $A, A^c := \boldsymbol{S} \setminus A \neq \emptyset$; a bounded step function $k : \boldsymbol{S} \to \mathbf{Z}_+$, i.e., $\sup_x k(x) < \infty$, and constants $\gamma, c \geq 0$, such that $\mathsf{E}_x\{f(\xi_{k(x)})\} \leq \exp\{-\gamma\} f(x)$, $x \notin A$; $\mathsf{E}_x\{f(\xi_{k(x)})\} < \infty$ for $x \in A$, and $\mathsf{E}_x\{f(\xi_1)\} \leq cf(x)$, $x \notin A$. If additionally, $A$ is a finite set and $f$ bounded away from 0, then that the stochastic process $r(X_n)$ converges exponentially quickly in $\mathsf{L}_1$ for any function $r$ bounded by $f$. The problem is how to compute explicit bounds.

In case of so-called stochastically monotone chains, Lund and Tweedie [1] have shown that the rate $\exp\{-\gamma\}$ in the Lyapunov function criterion is precisely the desired rate, provided $A$ consists of the 'minimal' state and the step function is identically equal to 1. They use a coupling time argument.

In general, even when one can contruct a contractive Lyapunov function, it maybe be difficult to meet the conditions required by the above authors. We will discuss some methods, what to do in those cases. We will illustrate these methods with some queueing examples and a stochastically non-monotone magneto-optical trap model.

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# Sequential Online Volatility Detection For Markov Modulated Asset Price Dynamics

William P. Malcolm

*National ICT Australia (NICTA), Systems Engineering and Complex Systems, Research School of Information Sciences and Engineering, The Australian National University Canberra ACT 0200 Australia* [paul.malcolm@nicta.com]

Robert Elliott

*Haskayne School of Business, University of Calgary, Calgary, Alberta, Canada T2N 1N4* [relliott@ucalgary.ca]

A fundamental task in financial modeling is that of calibrating asset price models, that is, estimating the parameters of mean return and volatility. Common techniques used to estimate these parameters are based upon maximum likelihood estimation, such as the Expectation Maximisation (EM) algorithm and its variants. These methods can be slow to converge and computationally intensive. Further, in practice, precise estimates of volatilities are often not necessary. For example, in option pricing with Markov modulated volatility models, a 10% error in volatility estimation is usually taken as acceptable.

In this article we suppose that one of $M$ candidate volalility models best explains a given asset price process. Sequential estimators are computed for each of the $M$ candidate models. These schemes compute an estimate for the relative likelihood of a given model explaining an observation process. Two classes of model are considered. In the first model, volatility states are determined by a continuous-time Markov chain. An important practical feature of the detection schemes we compute for this model, is that they do not inlcude stochastic inegration. Here we develop a version of the Clark Transformation [1] based on a Hadamard product, resulting in detector dynamics where the observation process appears as a parameter, rather than an integrator. In the second model, volatility states are determined by a discrete-time Markov chain. Computer simulations for the discrete-time Markov chain models are given on real data for the price evolution of West Texas Crude. These results are compared again an EM algorithm computation for the same data set.

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2. Aggoun, L. and Elliott R. J., Measure Theory and Filtering Cambridge University Press, 2004.

# On the existence of non-constant volatility in the Bachelier and Black-Scholes formulae

Kais Hamza

*School of Mathematical Sciences, Monash University 3800, Australia*
[Kais.Hamza@sci.monash.edu.au]

Fima Klebaner

*School of Mathematical Sciences, Monash University 3800, Australia*
[Fima.Klebaner@sci.monash.edu.au]

This paper looks at the existence of non-constant volatilities that agree with the Bachelier and Black-Scholes formulae for all strikes. More specifically, we ask the question of whether one can find a volatility process $\theta_t$ such that

$$\forall K, \qquad \mathbf{E}[(S_T - K)^+ | \mathcal{F}_t] = C(T, t, K, \theta_t, S_t),$$

where $C(T, t, K, \sigma, z)$ denotes the expressions for the call options for the Bachelier and Black-Scholes models respectively.

# Volatility Smile/Smirk Properties of [GLP & MEMM] Pricing Models

Yoshio Miyahara

*Graduate School of Economics, Nagoya City University, Yamanohata Mizuhochou Mizuhoku, Nagoya 467-8501, Japan*
[y-miya@econ.nagoya-cu.ac.jp]

Naruhiko Moriwaki

*Graduate School of Economics, Nagoya City University, Yamanohata Mizuhochou Mizuhoku, Nagoya 467-8501, Japan*

It is well-known that the implied volatility has smile or smirk properties in the real markets, and this property is called the volatility smile/smirk (or smile/skew) property. This fact suggests us the necessity of the construction of a new option pricing model other than the Black-Scholes model. Many kinds of models have been proposed and investigated. The [GLP & MEMM] (=Geometric Lévy Process & Minimal Entropy Martingale Measure) pricing model is one of them, and it is known that this model have many good properties as an option pricing model for the incomplete market. (See [1][2] [3]).

In this paper we investigate the volatility smile/smirk properties of the [GLP & MEMM] pricing models by the use of computer simulation method. We first explain the [GLP & MEMM] model briefly and give sevral examples of it. Next we calculate the implied volatility surface, and we see that the [GLP & MEMM] pricing model possesses this properties in various forms.

The results of this paper show us that the [GLP & MEMM] pricing model is a very strong candidate for the new model which has the volatility smile/smirk property.

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# Multivariate diffusion modelling

Michael Sørensen

*Department of Applied Mathematics and Statistics, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark*
[michael@math.ku.dk]

Tractable classes of multivariate diffusions are presented. For any given symmetric multivariate distribution, a diffusion process with linear drift and with marginal distribution equal to the given distribution is constructed. In particular, an expression for the diffusion matrix is given. In many examples this expression is explicit, and an approximation of the saddle-point type is given for use when it is not. The theory is particularly simple for normal variance-mixtures such as the symmetric generalized hyperbolic distributions; for an introduction to this class of distributions, see e.g. [2]. As a particular case, multivariate $t$-diffusions are considered.

Superposition of such multivariate diffusions result in a very flexible and tractable class of multivariate processes that generalize the one-dimensional models presented in [1]. Again any symmetric distribution can be obtained as the marginal distribution.

As a byproduct, one-dimensional processes can be obtained with a more general autocorrelation structure than the processes in [1]. In particular, a negative autocorrelation is possible. One application to finance of the theory presented here is to model the volatility process by means of a one-dimensional process of this type, and thus obtain a very flexible class of stochastic volatility models.

This is joint work with Martin Jacobsen, University of Copenhagen.

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2. Bibby, B.M. and Sørensen, M. (2003). Hyperbolic processes in finance. In: S. Rachev (Ed.): *Handbook of Heavy Tailed Distributions in Finance.* Elsevier Science, Amsterdam, 211–248.

# On stochastic processes in random environment and related topics

Kiyoshi Kawazu

*Department of Mathematics, Yamaguchi University, Yoshida 1677-1, Yamaguchi 753–8531, Japan* [kawazu@yamaguchi-u.ac.jp]

It is well known that there is a correspondence between random walks in random environment in the sense of Solomon and stochastic processes in random environment in one-dimensional space. That is,

$$\text{RW in RE} \Leftrightarrow dX_t = dB_t - \frac{1}{2}\{W'(X_t) + \kappa\}dt, \quad \kappa \text{ is a constant.}$$

In particular, the case $\kappa = 0$ (we call it a Brox diffusion, [1]) corresponds to Sinai's random walk [8] (recurrent case). We know their asymptotic behaviors are same and are of $1/(\log x)^2$-order. Its limit distribution is calculated by Golosov [2] and Kesten [7].

If the random environment is changed to one-sided Brownian motion, then we can obtain drastically strange phenomena [5, 6].

In high dimensional space, the papers Kalikow [4] and Zetouni [12] are well-known. In diffusion type cases, we have Tanaka's paper [10], which depends on Ichihara [3]. We try to consider the recurrence of product processes using Tomisaki's result [11].

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# On the Approximation of Jump-Diffusion Processes

Nicola Bruti-Liberati

*University of Technology Sydney, School of Finance & Economics, PO Box 123, Broadway, NSW, 2007, Australia* [Nicola.BrutiLiberati@uts.edu.au]

In finance key quantities are often described by stochastic differential equations (SDEs) of jump-diffusion type. The class of jump-diffusion SDEs that admits explicit solutions is rather limited. Consequently, there is a need for the systematic use of discrete time approximations in corresponding simulations. We propose several strong and weak numerical schemes for SDEs of jump-diffusion type. The strong schemes provide pathwise approximations and therefore can be employed in scenario analysis, filtering or hedge simulation. The weak schemes are appropriate for problems such as derivative pricing and evaluation of risk measures, where only an approximation of the probability distribution of the jump-diffusion process is needed. We provide some convergence theorems for the construction of approximations of any given order of convergence $\gamma \in \{0.5, 1, \ldots\}$ for SDEs driven by Wiener processes and Poisson random measures. We consider also derivative free, implicit and jump adapted approximations. For the commutative case particular schemes are obtained. Finally, a numerical study on the accuracy of the proposed schemes will be presented.

Joint work Eckhard Platen

## Stochastic Market Volatility Models

Truc Le

*School of Mathematical Sciences Building 28M, Monash University, Clayton Campus, Victoria 3800, Australia. Tel: (03) 9905 9760*
[truc.le@sci.monash.edu.au]

In this paper, we offer a new market-based approach to evaluating options on an asset. Our model corresponds to the real situations encountered in the market: option prices are not uniquely determined by their underlying asset but mainly by another factor, namely stochastic market volatility (or simply SMV). To begin constructing SMV, we assume that there exists a hedging portfolio which replicates perfectly the value of the underlying option. By ‘perfectly’, we mean that the value of the hedging portfolio will always equal exactly to the option. The hedging portfolio takes asset price and SMV as its input, therefore, for a given asset price the correct value of SMV gives the correct value for the option. SMV presents the dynamics of options market. We provide the proof of existence and uniqueness of solutions for SMV.

For the full paper, see *Applied Economics Letters* (2005), V.1, 3, pp.177–188.

# Measuring, pricing and hedging financial risk in a dynamic world

Nicole El Karoui

*CMAP, Ecole Polytechnique, Paris, France*
[elkaroui@cmapx.polytechnique.fr]

A large number of strategies are proposed on financial markets to control risks induced by market fluctuations. Traditional or more sophisticated (exotic) financial products are used by businesses or investors to transfer their risks to financial institutions.

Important questions faced by the market risk industry, e.g the development of asset pricing models and hedging strategies based on daily (infinitesimal) risk management criteria, have found answers through probabilistic tools and concepts, such as Brownian motion, martingales, and stochastic control. This in turn has had an important impact on the - exponential - development of this industry.

Numerical implementation is the cornerstone of the modelling process. The choice of a model is hence driven not only by its theoretical properties but also, and in great part, by its numerical tractability. This includes, among other things, reliable estimation of parameters, which are inferred from observable market data (financial products prices). Over the last few years, techniques for stabilizing these ill-posed problems via fast and accurate algorithms in partial differential equations have been developed.

In other respects, with motivation coming from multidimensional problems, Monte Carlo methods have been revisited in order to obtain accurate numerical results for prices of high-dimensional products or their derivatives with respect to key parameters. Efficient methods are based on differentiation on Wiener space and Malliavin calculus. A recent topic is that of solving optimization problems (optimal stopping times, optimal portfolio) through Monte Carlo methods.

Thanks in part to the size reached by derivatives markets, market authorities now require financial institutions to compute their daily global exposure (Value at Risk) via their own “internal” models. Motivated by this challenge, academic and risk-managers are debating the “best concept” of risk measure and various problems induced by the high-dimensional character of the covariance matrices they deal with.

Mathematical finance is a challenging and fast evolving domain, constantly looking for new ideas and concepts in Mathematics. One of its very remarkable aspects is the ability of theoretical research to have direct implications on daily market practice.

# Analysis of algorithms by the contraction method

Ludger Rüschendorf

*University of Freiburg, Germany* [ruschen@stochastik.uni-freiburg.de]

Stochastic recursive equations and algorithms arise in a great variety of problems originating from computer science like in algorithms of divide and conquer type but also in the probabilistic analysis of combinatorial optimization problems and in many other problems having a recursive nature. In some recent work quite general limit theorems for recursive algorithms have been obtained by the contraction method. By this method the limiting distribution of the algorithm is characterized as unique solution of some related stochastic fixpoint equation. The main result states that some information on the asymptotic of the first moment(s) implies together with the recursive structure a limit theorem for the algorithm. In this talk a survey is given on these developments and the contraction method is demonstrated at a series of examples.

# Nonreversible perturbations accelerate convergence

Chii-Ruey Hwang and Shuenn-Jyi Sheu

*Institute of Mathematics, Academia Sinica, Taipei, Taiwan 11529*
[crhwang@sinica.edu.tw]

Ma Shu-Yin Hwang

*Department of Business Mathematics, Soochow University, Taipei, Taiwan 10001*

To sample from distributions in high dimensional spaces or finite large sets directly is not feasible in practice, especially when the corresponding densities are known up to normalizing constants only. One has to resort to approximations. A Markov process with the underlying distribution as its equilibrium is often used to generate an approximation ("MCMC"). How good the approximation is depends on the approximating Markov process and on the specific criterion used for comparison. One may investigate the convergence properties of some particular Monte Carlo Markov processes, or compare the convergence rate within a family of Markov processes (with the same equilibrium) w.r.t. different criteria, or even try to find optimal solutions in that family. Mathematical problems arising from this approach are challenging. We prove, [1], that by simply adding a weighted divergence-free drift to a reversible diffusion, the convergence to equilibrium is accelerated. In other words, from an algorithmic point of view the nonreversible algorithm performs better. Note that different criteria are considered. The analysis is related to the study of antisymmetric perturbations of self-adjoint infinitesimal generators. Related problems will be discussed. For example the optimal solution is still open. A simulation study for two dimensional torus indicates that the rate could be infinite [2]. As for finite sample space, some preliminary results show that nonreversible perturbations accelerate convergence too.

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# Large deviations for random walks with regular exponentially decaying distributions

Konstantin A. Borovkov

*Department of Mathematics and Statistics, The University of Melbourne, Parkville 3010, Australia* [kostya@ms.unimelb.edu.au]

Alexander A. Borovkov

*Sobolev Institute of Mathematics, Novosibirsk 630090, Russia* [albert@sib.com]

We establish first order approximations and asymptotic expansions for probabilities of crossing arbitrary curvilinear boundaries in the large deviations range by random walks whose distribution tails differ from an exponential function by an integrable regularly varying factor: i.i.d. jumps $\xi_i$ in the walk have the right tails of the form

$$\mathbf{P}(\xi \geq t) = e^{-\lambda_+ t} V(t), \qquad \lambda_+ > 0,$$

where $V(t) = t^{-\alpha-1} L(t)$, $\alpha > 0$, $L$ is a slowly varying function at infinity.

In this interesting transient case, there exists a 'lower subzone' of the zone of large deviations, where the classical (Cramér) exact asymptotic results hold true, and an 'upper subzone', where only results on the crude logarithmic asymptotics were available. Now we derive exact asymptotic behaviour for the latter subzone and show that it is, in a sense, close to that described in the paper [1] where we dealt with regularly varying distribution tails. [Part of the presented results appeared in [2]]

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# Two strikes and you're out

Tom Cottrell and Robert Elliott

*Haskayne School of Business, University of Calgary, 2500 University Dr. NW, Calgary, AB T2N1N4, Canada* [cottrell@ucalgary.ca, relliott@ucalgary.ca]

There are many economic contexts in which the cost of making an irreversible investment may switch between a discrete set of strike prices determined by the state of some external variable. The changing state of the external environment is modelled by a Markov chain with regime-switching: see [1] and [2]. The optimal investment decision is modelled as a perpetual American option with a fluctuating strike. We determine the optimal investment policy in this regime-switching context.

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# Compound Poisson approximation via Stein's method

Aihua Xia

*Department of Mathematics and Statistics, the University of Melbourne, VIC 3010, Australia* [xia@ms.unimelb.edu.au]

In dependent systems, rare events have a tendency to appear in clusters which make Poisson distribution a less favourable model as approximation errors are too large to use. For such situations, a compound Poisson distribution seems to be a more suitable choice. Stein's method for the compound Poisson approximation was first introduced in Barbour, Chen and Loh (1992) but the approach yields relatively useful estimates for approximation errors only when the approximating compound Poisson distribution $Z = \sum_{j=1}^{\infty} jN_j$ satisfies $j\lambda_j \downarrow 0$ as $j \to \infty$, where $N_j \sim \text{Poisson}(\lambda_j)$, $j \geq 1$ are independent, because under this condition, a Markov immigration-death process with multiple births and unit per capita death rate can be brought in to estimate the Stein factors. In this talk, we present a Stein's equation for compound Poisson approximation using immigration-death processes with multiple births and multiple deaths, and use it to estimate the total variation distance between a compound Poisson distribution and the distribution of the sum of independent integer-valued random variables.

# On optimality condition of complex systems

VICTOR KOROTKIKH AND GALINA KOROTKIKH

*Faculty of Informatics and Communication, Central Queensland University, Mackay, 4740, Queensland, Australia* [v.korotkikh@cqu.edu.au g.korotkikh@cqu.edu.au]

The efficient management of complex systems is becoming increasingly important. However, despite significant progress and interest in complex systems, there is a limited understanding of the problem. In particular, because the existence of principles governing the non-equilibrium situation has not yet been established [1], the possibility of a general condition determining the optimal performance of a complex system is still unknown.

To contribute in this direction, an optimization algorithm as a complex system is presented. The performance of the algorithm for any problem is controlled as a convex function with a single optimum. To characterize the performance optimums, certain quantities of the algorithm and the problem are suggested and interpreted as their complexities [2]. An optimality condition of the algorithm is computationally found: *if the algorithm shows its best performance for a problem, then the complexity of the algorithm is in a linear relationship with the complexity of the problem.*

The optimality condition provides a new perspective to the subject by recognizing that the relationship between certain quantities of the complex system and the problem may determine the optimal performance.

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# Factor distributions and correlations implied by market quotes for synthetic CDO tranches

Erik Schlögl

*School of Finance and Economics, University of Technology, Sydney, Broadway, NSW 2007, Australia.* [Erik.Schlogl@uts.edu.au]

Traditionally, default dependence has been the most difficult calibration issue in credit risk modelling. This is because there were no liquidly traded instruments from which a "market implied" correlation could be inferred. However, the rapid pace of innovation in the market for credit risk has recently given rise to new financial products, synthetic collateralised debt obligation (CDO) tranches, the prices of which will increasingly serve to aggregate the market views on default dependence between different obligors. Already, practitioners are talking about implied correlation "smiles" and "skews" in a manner reminiscent of the volatility smiles found in liquid option markets. On the one hand, "implied correlation" is a far more complicated concept than implied volatility, imposing limits as to how far this analogy can be taken when making relative value assessments about derivative financial instruments. However, there remain useful applications in robustly pricing bespoke tranches on standardised portfolios. In particular, one may represent the information about default correlation embedded in market quotes for synthetic CDO tranches by extracting implied factor distributions.

# An Examination of the Effect of Non-Normality on Optimal Portfolio Construction: A Copula Based Approach

Anthony Hatherley

*UQ Business School, The University of Queensland, Brisbane 4072, Australia*
[a.hatherley@business.uq.edu.au]

Tactical asset allocation decisions that are dependent on Markowitz's Mean Variance Portfolio Theory (MVPT) rely heavily on assumptions of normality. Asset returns are assumed to be normally distributed, whilst the dependence structure between assets follows a multivariate normal distribution.

A well known empirical result is that the returns of many asset classes are non-Gaussian [1]. This raises concerns over the appropriateness of the use of MVPT for the purpose of portfolio selection. In particular, MVPT is used to determine the optimal portfolio choice given either the expected return or standard deviation of returns [2]. Furthermore, the dependence structure between assets, given by the correlation matrix, is multivariate normal. The choice of portfolio under this setting does not take into account other possible characteristics of the return distribution (such as skewness or kurtosis) or alternative relationships between assets (such as tail dependence) [3].

The aim of this project is to determine the significance of non-Gaussian assumptions on portfolio selection and optimization across a small, but representative set of asset indices in the context of Tactical Asset Allocation. In particular if non-normal assumptions produce a more efficient frontier, how inefficient are portfolios based on normally distributed returns? What additional risk do fund managers incur by assuming normally distributed returns and dependence structures?

This investigation relies on the application of a class of mathematical functions, known as copula functions, in describing the dependence structure between assets. Copula functions allow for a joint distribution to be constructed using only the observed marginal distributions of the individual assets, and can incorporate a broad range of non-Gaussian dependency structures. In the context of this investigation, it follows that the user is not constrained in their choice of distribution function for asset returns.

Therefore, non-Gaussian distribution functions may be specified for the return distribution of individual assets (these distribution functions need not be the same). Next, the choice of copula can be used to represent different characteristics between assets that are typically not captured by the covariance matrix (such as "fat tails" or tail dependence). The resultant efficient frontiers produced under non-normality (with possible concordance interrelations) can then be statistically compared to frontiers generated under MVPT.

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# A comparative study of hedge performance robustness for equity index models

Hardy Hulley

*School of Finance and Economics, University of Technology, Sydney, P.O. Box 123, Broadway, NSW 2007, Australia* [hardy.hulley@uts.edu.au]

David Heath and Eckhard Platen

*Department of Mathematical Sciences & School of Finance and Economics, University of Technology, Sydney, P.O. Box 123, Broadway, NSW 2007, Australia*

The subject of this paper is a comparative empirical study of three alternative stochastic models for equity index dynamics: *geometric Brownian motion*; the *modified constant elasticity of variance model*; and a *minimal market model*. Each model is described by a single parameter family of stochastic differential equations, and each admits an option pricing formula where the parameter appears as a free variable. Using historical data, we simulate the hedge portfolios arising from the three models in question, for European call options on the S&P500 index. For each simulation the model parameters are chosen to optimize the cost of hedging an at-the-money call over the simulation period. We examine the robustness of hedge performance with respect to option strike and maturity. This analysis is applied to the problem of determining which model best fits S&P500 index data.

# A Complex Systems Approach to Spatial Epidemics

Joana M. Simoes

*Centre for Advanced Spatial Analysis, University College of London, 1-19 Torrington Place, Gower Street London WC1E 6BT, UK*
[j.simoes@ucl.ac.uk]

Spatial Epidemics concern the analysis of the spatial distribution of a disease [1]. In human epidemics, the spread of infectious diseases is highly influenced by the structure of the underlying social network [2]
The target of this study is not the network of acquaintances, but the social mobility network: the daily movement of people between locations, in cities, which has already been described as a small world network [3]. This research led to the implementing of a agent based model (ABM), that comprehends both a movement and a infection model.
The movement model establishes the mobility network, which is inherently spacial, and is implemented using Geographical Information Systems (GIS).
Using a bottom up approach, the global structure of the network is emerged from the displacement of each individual ($a$), according to the expression:

$$a_{(i,j)}^{(t+1)} = a_{(i,j)}^{(t)} + d, \tag{1}$$

The stochastic variable $d$ has a probabilistic distribution, according to:

$$d = (P_1)D_1 + (P_2)D_2 + (P_3)D_3 + (P_4)D_4 = \sum_{4}^{x=1}(P_x)D_x = 1, \tag{2}$$

D1, D2, D3 and D4 are the different ranges of movement.
The infection model describes the contagious process in the network established in the movement model.
The topology of the network is regenerated at each time step, and a evolutionary virus is simulated using random mutations on the infection force.
In this paper, the model will be described, and it will be shown a sensitivity analysis to evaluate the influence of the different parameters and understand a bit of its mechanics. Finally, it will be shown a aplication on a dataset of a mumps epidemic (Portugal, 1993-1996). The results will be discussed and some conclusions will be drawn.

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# Multivariate heavy tails, asymptotic independence and beyond

Sidney Resnick

*School of Operations Research and Industrial Engineering, Rhodes Hall 284, Cornell University, Ithaca, NY 14853 USA* [sir1@cornell.edu]

A random vector having a distribution which is multivariate regularly varying at infinity can have a dependence structure which is hard to specify in practice. One extreme but not uncommon case is "asymptotic independence" which roughly describes the situation where the random vector's components are not simultaneously large. In the absence of fruther assumptions, estimation of the probability of extreme risk sets yields estimates which are null. One way to remedy this is through hidden regular variation [5, 7, 2, 6] which measures variables on a different scale. Another is via conditioning on one component being large and using a limiting distribution as the conditioning variable is pushed to infinity [4, 1, 3]. We discuss detection of hidden regular variation along with other extensions into conditional models. An application to network data is provided. (Portions are joint with J. Heffernan, Lancaster, UK.)

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# Using the fossil record to date splits in the primate tree

Simon Tavaré and Richard Wilkinson

*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, England*
s.tavare@damtp.cam.ac.uk,rdw27@cam.ac.uk

Inference about the divergence times of species has long been of interest to biologists. Molecular evolutionists usually date such splits using DNA sequence data [1, 2], while paleontologists use a literal reading of the fossil record for this purpose [3]. It is common that estimates derived from these approaches differ substantially, molecular estimates often being higher than the fossil record suggests.

In an attempt to resolve these differences, an alternative method of inferring the divergence time of a group of species using data from the fossil record was presented in [4]. The method requires a model for species divergence, and uses as data the number of species found as fossils in a series of stratigraphic intervals. In this talk we present an approximate Bayesian computation approach that finds the posterior distribution of the temporal gap (the time from the oldest known fossil in the group to the point of divergence of the group) and also dates the split time of internal nodes of interest. The method, which readily allows a comparison among different evolutionary scenarios, is illustrated using data from the primate fossil record.

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# A Free Boundary Problem Related to Environmental Management System

Robert Elliott

*Haskayne School of Business, University of Calgary, Calgary, Alberta, Canada T2N 1N4* [relliott@ucalgary.ca]

Alexei Filinkov

*Land Operations Division, Defence Science and Technology Organisation, Edinburgh SA 5111, Australia & School of Mathematical Sciences, University of Adelaide, Adelaide SA 5005, Australia* [alexei.filinkov@adelaide.edu.au]

Robin Nicholson

*Land Operations Division, Defence Science and Technology Organisation, Edinburgh SA 5111, Australia* [robin.nicholson@dsto.defence.gov.au]

In paper [1] Pindyck considers the optimal time to reduce the rate of pollution in an environmental model. He considers two processes, one of which $(S)$ represents the level of pollution or degradation in some environmental area. The second $(\theta)$ models the social, political or financial cost of the pollution. Pindyck considers a single control parameter which models reducing the rate of pollution. It is assumed that this reduction can be made only once and it cannot be reversed. This is justified by observing that such irreversibility is often inherent in environmental policy: once decisions have been made, they are not usually overturned.

Pindyck initially supposes that $\theta$ switches only at a known time $T$ to one of two values, $\underline{\theta}$ or $\overline{\theta}$, where $\underline{\theta} \leq \theta \leq \overline{\theta}$. More interestingly later in his paper he assumes $\theta$ is described by a log-normal process. However, only in a special case is an explicit solution is obtained.

In this paper we suppose the level of pollution is described by a log-normal process whose dynamics involve a control parameter $u$. We briefly discuss the relative advantages of switching the control when $\theta$ only jumps to $\overline{\theta}$ or $\underline{\theta}$ at time $T$. Proceeding to the case of log-normal dynamics for $\theta$ we can give an explicit solution of a free boundary problem which provides the optimal time to reduce $u_0$ to $u_1$.

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# Modelling a plantation-nursery system

Joe Gani

*Mathematical Sciences Institute, Australian National University, Canberra ACT 0200, Australia* [gani@maths.anu.edu.au]

A plantation is subject to insect-borne infections; infected trees are cut down and replaced by seedlings from an adjacent nursery. Unfortunately, some of these seedlings are also infected. The number of infected trees $X(t)$ in the plantation can be represented by a Markov chain in continuous time. We derive a set of Kolmogorov equations for the probabilities of $X(t)$, and solve these using Laplace transforms. The main result is that it takes a very long time for the process to reach its stationary state.

# Commodity Prices and Regime Switching Bases

Catherine Elliott

*Stochastics Inc, 31 Varsity Estates Park NW, Calgary Alberta Canada, T3A 6A4*

Robert J. Elliott

*Haskayne School of Business, University of Calgary, Calgary, Alberta, Canada T2N 1N4* [relliott@ucalgary.ca]

William P. Malcolm

*National ICT Australia (NICTA), Systems Engineering and Complex Systems, Research School of Information Sciences and Engineering, The Australian National University Canberra ACT 0200 Australia* [paul.malcolm@nicta.com]

In the natural gas market the basis is the difference in the price of gas at two delivery points. The usual reference in the U.S.A. for a basis differential is NYMEX. For example, if the May Henry Hub price is \$5.25 and the May NYMEX price is \$5.45 then the basis differential for May NYMEX is \$0.20 to Henry. The usual reference for Canada is the price at the AECO facility.

In this article we propose to model the basis as a mean reverting diffusion, $X = \{X_t, t \geq 0\}$. Unlike a price process the basis process $X$ can take positive or negative values. The new feature in our model is that we suppose the mean reverting level in our dynamics can change according to the state of the economy. The economy is modelled as a finite state Markov chain $Z = \{Z_t, t \geq 0\}$ and the economy can perhaps be in two states ('good' and 'bad'), or possibly three states.

Our continuous time model is discretized and the results of Elliott *et al* [1], are adapted to obtain a recursive filter for the state of the economy given observations of $X$. In turn, this allows predictions to be made of the basis at the next time. If the observed basis is then higher or lower than the predicted value, it suggests one price is possibly higher than it should be and the other lower. Consequently, a trading strategy can be implemented based on these predictions. Computer simulations are provided to demonstrate the benefit of the Markov modulated mean reverting model we propose and the estimation schemes developed to facilitate the trading strategies just described.

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# Stochastic Life Annuities

Daniel Dufresne

*Centre for Actuarial Studies, University of Melbourne, VIC 3010, Australia*
[dufresne@unimelb.edu.au]

A problem which had some vogue in actuarial circles in previous decades was to find the distribution of the amount required to fund a life annuity; this amount was called a "stochastic life annuity." The distribution of the stochastic life annuity may be used to answer questions such as "What is the probability that an amount $F$ is sufficient to fund a life pension with annual amount $y$ to a pensioner aged $x$, given that log-returns have mean $m$ and volatility $\sigma$?"

There are two sources of randomness, the survival of the pensioner, and the returns of the amount invested. I will assume that the amount invested evolves as a geometric Brownian motion, from which the annuity payments are deducted in a continuous fashion, and that the pensioner's duration of life is independent of the Brownian motion. The main ingredients of the solution given in the talk are the known results about the integral of geometric Brownian motion, and the approximation of probability distributions by combinations of exponentials.

# Valuing production capacities on electricity

Juri Hinz

*Institute for Operations Research, ETH Zentrum, CH–8092 Zurich, Switzerland*
[hinz@ifor.math.ethz.ch]

Price risk management for electrical power is not trivial due to restrictions on storability of the underlying. Moreover, agents face high risk complexity resulting from patterns in price seasonality, time-varying volatility, and high price spikes. To be protected against price risk, consumers purchase diverse swing–type contracts, whereas contract writers try to hedge them by appropriate physical assets, for instance, by storage utilities, by transmission and/or production capacities.

Due to difficulties in valuation and hedging of electricity derivatives, option writers prefer to sell agreements which are easily replicated by appropriate physical assets. As a result, we observe that many electricity derivatives are of swing type, presenting corresponding financial counterparts of agreements on production capacities. For example, a popular instrument is the *virtual production capacity* with strike price $K > 0$ and availability period $[0, \vartheta]$, which is an American-type contract where the holder can opt any exercise policy $(q_t)_{t\in[0,\vartheta]}$ obeying

$$0 \leq q_t \leq \lambda \text{ for all } t \in [0, \vartheta], \qquad \int_0^\vartheta q_t dt \leq \Lambda \tag{1}$$

whereas the writer is obliged to supply a cash–flow at intensity $(q_t(E_t - K)^+)_{t\in[0,\vartheta]}$ depending on electricity price $(E_t)_{t\in[0,\vartheta]}$. Obviously, this agreement is hedged by a real production unit available within $[0, \vartheta]$ with production costs $K$, maximal electrical power $\lambda > 0$, and total amount of energy $\Lambda > 0$. The correct valuation of such contacts is still under lively debate.

The major part of our approach deals with the concept of risk–neutral spot price dynamics. The difficulty here is that electricity spot prices at different times are not directly related to each other, strictly speaking, $E_s$ and $E_t$ are to consider as prices for different commodities delivered at different dates $s \neq t$. We suggest an axiomatic setting to discuss price dynamics for contracts on a flow commodity: (i) the price evolution is described by stochastic processes with appropriate path properties, (ii) the model explains the initial forward curve, (iii) it excludes arbitrage opportunities, and (iv) it reflects restrictions on storability of the underlying. It turns out such assumptions already provide a framework where the standard change–of–numeraire transformation converts a flow commodity market into a market consisting of zero bonds and some additional risky asset. Utilizing this structure, we apply the toolkit of interest rate theory to price the availability of production capacity on electricity.

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# Modelling and Analysing of Stochastic Failures in Complex Component-Based Systems

Lars Grunske

*University of Queensland, School of ITEE (Boeing Postdoctoral Research Fellow), 4072 Brisbane (St.Lucia), Australia* [grunske@itee.uq.edu.au ]

Complex computer-based systems used in mission- or safety-critical domains, including defence applications, air traffic control, railway signalling and medical applications play an important role in our modern society. One important task in the development of these systems is the construction of safe cases and safety models that are used to determine quantitative measures for failure or hazard probabilities. These safety models should be intuitive, compositional and have the expressive power to model both software and hardware behaviour.

In industrial projects, currently event-based models such as Fault Trees or state-based models such as Markov-chains are used. Each of these models has it limitations [3]. A model that combine elements from Fault Trees and Markov Models could improve expressive power of safety cases. In [2], we (Bernhard Kaiser, Yiannis Papadopoulos, and Lars Grunske) have introduced State Event Fault Trees (SEFTs), a new model for safety analysis with a combined state-event semantic. SEFTs are a hierarchical and visual model that integrates elements from stochastic state-based models (Markov-chains) with FTs. The quantitative probabilistic analysis is performed by translation of the safety models into Deterministic and Stochastic Petri Nets (DSPNs) [1], a class of Petri Nets for which analysis tools exist (e.g. the tool TimeNET [4]).

In the proposed talk I want to explore the problems and benefits of State Event Fault Trees in the construction of safety cases for complex computer-based systems. Furthermore, I will address the usefulness of strongly encapsulated and hierarchical evaluation models, such as SEFT, to analyse the stochastic behaviour of complex component-based systems.

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# A Mathematical Model for Opportunistic Timing and Manipulation in Australian Federal Elections

Dharma Lesmono

*Department of Mathematics, The University of Queensland, Brisbane, Qld 4072 AUSTRALIA* [dlesmono@maths.uq.edu.au]

In this paper we develop a mathematical model for election timing using stochastic dynamic programming and game theory approach. In many Majoritarian Parliamentary Systems, the government has a constitutional right to call an early election. This right can give the government an advantage to remain in power for as long as possible by calling an election when its popularity is high. This problem can be compared with the determination of early exercise for American options in finance. This election timing is considered as a zero-sum game between the government and the opposition. Our analysis is based on the two-party-preferred data which measure the popularity of the government and the opposition. We propose a Stochastic Differential Equation (SDE) to describe the behaviour of the poll process and use a Maximum Likelihood Estimation (MLE) method to estimate its parameters. We assume that the government can call an early election and use controls termed 'boosts' to raise its popularity in the poll by introducing policy or economic actions. On the other hand, the opposition can also use boosts to pull the government's popularity down by introducing policy and economic responses. Results are given in terms of the expected remaining life in power, call and boost probabilities at each time at any level of popularity. We are particularly interested in the Australian Federal Election for House of Representatives and perform a case study.

[This is a joint work with Elliot Tonkes and Kevin Burrage]

# A Quadratic Gaussian Reduced Form Model

Samson Assefa

*School of Finance and Economics, University of Technology, Sydney, PO Box 123, Broadway NSW 2007, Australia* [Samson.Assefa@student.uts.edu.au]

This paper considers the pricing of credit sensitive securities under a two country reduced form model. This framework enables us to consider the pricing of securities that are subject to quanto risk, where payoffs are denominated in a currency other than the currency of the underlying asset determining the payoff amount. We consider a multifactor model where the domestic and foreign interest rates as well as the intensity of default are modelled by quadratic Gaussian processes. We show how to calibrate to the term structure of interest rates and to credit default swaps. When the number of correlated factors is less or equal to two, we provide analytic formulas for the price of a credit default swap option and an approximation when a higher number of correlated factors are involved.

# Branching processes in random environment and the bottleneck of evolution

Vladimir Vatutin and Elena Dyakonova

*Department of Discrete Mathematics, Steklov Mathematical Institute, 8 Gubkin St., Moscow 19991, Russia* [vatutin@mi.ras.ru, elena@mi.ras.ru]

Let $Z_n$ be the number of particles at time $n = 1, 2, ...$, in a branching process in random environment specified by a sequence of iid probability generating functions $\{f_n(s)\}_{n\geq 1}$, $s \in [0,1]$. Set $X_k = \log f_k'(1)$, let $S_0 = 0, S_n = X_1 + \ldots + X_n$, $n \geq 1$, be the associated random walk for this process and denote by $\tau(n)$ the left-most point of minimum of $S_k, k = 0, 1, \ldots,$ on the interval $0 \leq k \leq n$. Assuming that Spitzer's condition

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} P(S_k > 0) \to \rho \in (0,1)$$

fulfills and imposing some additional mild conditions on the characteristics of $f_n(s)$, we proof (under the quenched approach) conditional limit theorems for the process $\{Z_{nt},\ t \in (0,1]\}$ conditioned on the event $\{Z_n > 0\}$. It is shown that up to a random multiplier, being positive and bounded with probability 1, $Z_{nt}$ grows like $e^{S_{nt} - S_{\tau(nt)}}$ as $n \to \infty$ and, therefore, is subject to large oscillations. This means, in particular, that the process passes during its evolution through a sequence of "bottlenecks" which coincide with the strict descending ladder epochs of $S_k$ within the interval $[0, n]$.

We also study the distribution of the distance to the closest mutual ancestor of the individuals of the $n$-th generation and show that the closest mutual ancestor is located in a vicinity of point $\tau(n)$.

**Acknowledgment.** This work was supported in part by the INTAS grant 03-51-5018, by the joint NWO-RFBR grant 047.016.013 and by the program "Contemporary problems of Fundamental Mathematics" of the Russian Academy of Sciences.

# Models of financial markets with asymmetric information: additional utility and entropy of information

Peter Imkeller

*Department of Mathematics, Humboldt University Berlin, Unter den Linden 6, 10099 Berlin, Germany* [imkeller@math.hu-berlin.de]

We consider models of financial markets with agents on different information levels: *non-informed traders* whose information follows the natural evolution of the underlying price processes, and *insiders* who possess some extra information which is completely revealed to the non-informed traders only at the end of the trading interval. Relevant additional information may be given by some knowledge about the price at a later time, about the maximal price, or the last passage of a certain level by the price process. We show that the expected additional logarithmic utility of an insider is given by the entropy of the additional information in a quite general framework, and study similar notions for different utility functions. We discuss the existence of equivalent martingale measures, and explain how the semimartingale property of price dynamics is linked to properties of the *information drift*. We investigate the problem, how additional information may be blurred to rule out arbitrage. Our techniques are embedded in general semimartingale theory, include *grossissement de filtrations* methods, and necessary extensions involving notions of *Malliavin's calculus*.

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# On the Role of the Growth Optimal Portfolio

Eckhard Platen

*University of Technology Sydney, School of Finance & Economics and Department of Mathematical Sciences, PO Box 123, Broadway, NSW, 2007, Australia* [eckhard.platen@uts.edu.au]

The paper discusses various roles that the growth optimal portfolio (GOP) plays in finance, see [1], [2]. For the case of a continuous market we show how the GOP can be interpreted as a fundamental building block in financial market modeling, portfolio optimization, contingent claim pricing and risk measurement. On the basis of a portfolio selection theorem, optimal portfolios are derived. These allocate funds into the GOP and the savings account. A risk aversion coefficient is introduced, controlling the amount invested in the savings account, which allows to characterize portfolio strategies that maximize expected utilities. Natural conditions are formulated under which the GOP appears as the market portfolio. A derivation of the intertemporal capital asset pricing model is given without relying on Markovianity, equilibrium arguments or utility functions. Fair contingent claim pricing, with the GOP as numeraire portfolio, is shown to generalize risk neutral and actuarial pricing. Finally, the GOP is described in various ways as the best performing portfolio.

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# Optimal logarithmic utility for insiders in Lévy market

Makoto Yamazato

*Department of Mathematical Sciences, Faculty of Science, University of the Ryukyus, Nishihara-cho, Okinawa 903-0213, Japan*
[yamazato@math.u-ryukyu.ac.jp]

Arturo Kohatsu-Higa

*Department of Economics, Universitat Pompeu Fabra, Ramón Trias Fargas*
[kohatsu@upf.es]

Let $L$ be a 1-dimensional Lévy process. Suppose that the discounted stock price of a stock is given by

$$\widehat{S}_t = S_0 \exp\Big((b - r - \frac{c^2}{2})t + L_t\Big), \quad 0 \le t \le T.$$

We regard insider's knowledge as an enlargement of filtration following Karatzas-Pikovsky ([2]). Normal invester's filtration is $\{\mathcal{F}_t = \sigma(L_s, s \le t)\}_{0 \le t \le T}$. Let $H$ be a 1-dimensional Lévy process independent of $L$. We define insider's filtration by $\{\mathcal{G}_t = \mathcal{F}_t \vee \sigma(L_T + H_{(T-s)^\alpha}; s \le t)\}_{0 \le t \le T}$, $0 < \alpha < 1$. So, the insider's portfolio $\{\pi_t\}$ is assumed to be $\{\mathcal{G}_t\}$-predictable. This means that the insider knows the maturity price of the stock which is disturbed by the independent progressive noise $H$.

We discuss in this talk whether the optimal logarithmic utility $\max_\pi E(\log \widehat{V}_T)$ is finite, where $\widehat{V}_t$ is the discounted wealth process defined by the equation $\widehat{V}_t = V_0 + \int_0^t \frac{\pi_{s-}\widehat{V}_{s-}}{\widehat{S}_{s-}} d\widehat{S}_s$. While in Brownian motion case the optimal logarithmic utility is finite ([1],[3]), in this case the result is very complicated.

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# Rank Process and Stochastic Corridor

Ryozo Miura

*Graduate School of International Corporate Strategy, Hitotsubashi University, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8439, Japan*
[rmiura@ics.hit-u.ac.jp]

Fujita and Miura (2004) defined a rank statistics of a continuous time stock price process and derived its probability distribution under the assumption that the stock price follow a Geometric Brownian motion. The present paper uses rank statistics to define a new exotic derivatives; stochastic corridor. The stochastic corridor based on rank statistic measures how many days during the prefixed time interval the stock prices stay below the price of a prefixed day t. The special feature of the rank statistics is that its distribution does not depend on the stock price at the beginning of the prefixed time interval. Swap or exchange contract and option with spot starting corridor and forward starting corridor will be defined and their pricing will also be discussed.

# Optimal control of stochastic differential delay equations with application in economics

ANATOLI F. IVANOV

*Department of Mathematics, Pennsylvania State University, P.O.Box PSU, Lehman, PA 18708, USA* [afi1@psu.edu]

ANATOLY V. SWISHCHUK

*Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta T2N 1N4, Canada* [aswish@math.ucalgary.ca]

This research deals with the study of optimal control of stochastic differential delay equations and their applications. By using the Dynkin formula and solution of the Dirichlet-Poisson problem, the Hamilton-Jacobi-Bellman (HJB) equation and the inverse HJB equation are derived. Application is given to a stochastic model in economics, a Ramsey model with delay and randomness .

The model is described by the equation

$$dK(t) = [BK(t-T) - u(K(t))C(t)]\,dt + \sigma(K(t-T))dw(t)$$

where $K$ is the capital, $C$ is the production rate, $u$ is a control process, $B$ is a positive constant, $\sigma$ is a standard deviation of the "noise". The "initial capital"

$$K(t) = \phi(t), \quad t \in [-T, 0],$$

is a continuous bounded positive function. For this stochastic economic model the optimal control is found to be $u_{\min} = K(0) \cdot C(0)$, and the optimal performance is

$$\begin{aligned} J(K, u_{\min}) &= \tfrac{K^2(0)}{2} + \tfrac{K^2(0)\cdot C^2(0)}{2} + \textstyle\int_{-T}^{0} \phi^2(\theta)\,d\theta = \\ &= \tfrac{K^2(0)}{2}(1 + C^2(0)) + \textstyle\int_{-T}^{0} \phi^2(\theta)\,d\theta. \end{aligned}$$

The full version of this paper is submitted for publication [1]. Necessary preliminaries on stochastic differential equations and the original Ramsey model can be found in [2] and [3], respectively.

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# Using Bellman's Principle without the Bellman Equation: New Parallel-Computing-Capable Numerical Methods for Optimal Stopping, with Applications to Financial Economics in View

Andrew Lyasoff

*Department of Mathematics and Statistics, Boston University, 111 Cummington St., Boston, MA, 02215, USA* [alyasoff@bu.edu]

Many optimal stopping problems – especially the ones encountered in financial economics – essentially come down to constructing a function $(t, x) \longrightarrow u(t, x)$ that has the property:

$$u(t,x) = \max[\Lambda(x), e^{-r\,\delta}\mathbf{E}[u(t+\delta, X^x_\delta)]] + o(\delta)\,, \tag{1}$$

where $\Lambda(\cdot)$ is the termination payoff function and $(X^x_t)_{t\geq 0}$ is some Markov process governed by some stochastic equation of the form

$$X^x_t = x + \int_0^t \sigma(X_s)dW_s + \int_0^t a(X_s)ds\,. \tag{2}$$

Traditionally, the function $u(\cdot,\cdot)$ has been computed by way of solving the associated Bellman equation, for which various numerical techniques – mostly variations of the finite difference scheme – have been developed. A new approach, which takes advantage of the recent developments in computing technology and allows one to construct the function $u(\cdot,\cdot)$ directly, i.e., without any reference to the Bellman equation, by way of backward induction governed by Bellman's principle (1), is described in [1]. In this approach, equation (2) is approximated by an equation with affine coefficients which admits an "explicit" solution in terms of integrals of the exponential Brownian motion. The expectation in the right side of (1) is calculated in [1] by using a rather crude approximation of the distribution of the *integral of the exponential Brownian motion* (IEBM). In this paper various methods for computing integrals involving the probability density function of the IEBM will be discussed. While our main interest in such calculations is motivated by optimal stopping problems and the general procedure described in [1], the distribution of the IEBM has been of particular interest in mathematical finance in connection with the so called Asian options, as explained in [2] and [3]. New methods for computing the function $u(\cdot,\cdot)$ directly from Bellman's principle (1) will be presented and the parallel computing aspects of such procedures will be discussed.

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# Palm distributions and invariance properties of spatial point processes

Günter Last

*Institut für Mathematische Stochastik, Universität Karlsruhe (TH), 76128 Karlsruhe, Germany* [last@math.uni-karlsruhe.de]

It is a fundamental and classical fact (see e.g. [1]), that there is an essentially unique correspondence between stationary point processes on the line and stationary sequences of interpoint distances. A spatial version of this result is not at all obvious. Thorisson ([4]) proposed to use *bijective point shifts* to characterize Palm distributions of stationary point processes. We will present the main result in [2] showing that invariance under such point shifts does indeed provide an intrinsic description of Palm distributions. The proof is based on *symmetric area search* and on Mecke's [3] intrinsic characterization of Palm measures.

Timar [5] found a one-ended tree on stationary point processes that is constructed in a shift-invariant way. This amazing result suggests that it might be possible to identify Palm distributions as the probability measures that are invariant under a rather small family of bijective point shifts. We will discuss Timar's result as well as related questions on graphs and trees supported by stationary point processes.

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# Eigenfunction methods for estimation with random fields

Christopher G. Small

*Department of Statistics and Actuarial Science, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, Canada N2L 3G1*
[cgsmall@uwaterloo.ca]

It is a well accepted that optimal inference about the mean of a distribution requires some knowledge of its dispersion and shape. For a continuous-time stochastic process or for a random field, fully optimal inference about the mean function requires knowledge about the **covariance kernel**, which is the limiting form of the covariance matrix for data in finite dimensions. Suppose that $X(\mathbf{t})$ is a real-valued stochastic process or field, where $\mathbf{t} \in I\!R^q$. In practice $X(\mathbf{t})$ will only be observed for $\mathbf{t}$ within some bounded "window." For the purposes of this paper, we shall assume that $\mathbf{t}$ lies in some bounded closed subset $\mathcal{R}$ of $I\!R^q$ with nonempty interior.

$$E_\theta[X(\mathbf{t})] = \mu_\theta(\mathbf{t}) \text{ and } \quad \mathrm{Cov}_\theta[X(\mathbf{s}), X(\mathbf{t})] = \delta\, \Gamma_\theta(\mathbf{s}, \mathbf{t})$$

be the mean function and covariance kernel respectively, where $\mathbf{s}$, $\mathbf{t} \in \mathcal{R}$. We let $\theta$ denote a $k$-dimensional column vector of parameters. Both $\mu_\theta$ and $\Gamma_\theta$ are assumed to be known real-valued functions of the unknown parameter $\theta$. The quantity $\delta > 0$ is assumed to be unknown, or known and equal to to one for some models, but functionally free of $\theta$.

To estimate $\theta$ or $\mu(\theta)$, the Karhunen-Loève expansion provides a useful decomposition of the process into eigenfunction for the covariance kernel. This leads to an estimating equation for $\theta$ of the form

$$\sum_{j=1}^{\infty} \rho_j^{-1}(\theta)\tau_j(\theta)[Y_j(\theta) - \nu_j(\theta)] = 0,$$

where $\rho_j$, $\tau_j$ and $\nu_j$ arise from the K.-L. expansion, and play the role of the mean gradient, the variance, and the mean of $Y - j$ respective. The formulation reduces to the classic quasi-likelihood (generalised estimating equation) setting.

This elegant solution to the estimation problem masks a number of practical and theoretic difficulties. For many data sets the covariance kernel must be estimated. But this is problematic if stationarity cannot be assumed. Secondly, the actual eigenfunction decomposition is nontrivial to compute for many kernels. In this talk, I shall consider a family of "working kernels" analogous to the working covariance matrices of quasi-likelihood or generalised estimating equations.

# Pricing discretely monitored exotic options under the Levy process framework

Alexander Novikov

*Department of Mathematical Sciences, University of Technology, Sydney, Broadway, PO Box 123 NSW 2007 Australia* [Alex.Novikov@uts.edu.au]

Konstantin Borovkov

*Department of Mathematics and Statistics, The University of Melbourne, Parkville, VIC, 3010 Australia* [K.Borovkov@ms.unimelb.edu.au]

We present bounds for the rates of convergence of the prices of discretely monitored exotic options (of barrier and lookback types) to those of the continuous ones when the number of observations goes to infinity. We consider the Geometric Lévy process framework with time-dependent parameters for the underlying stock price. Based on these bounds, we construct numerical procedures for interpolating curves. For the case of the Black-Scholes model with a constant interest rate, we suggest second order approximations as well.

# Analytic Pricing of European Contingent Claims under the Real World Measure

Shane M. Miller

*School of Finance and Economics & Department of Mathematics, University of Technology, Sydney, Australia.* [Shane.M.Miller@.uts.edu.au]

Eckhard Platen

*School of Finance and Economics & Department of Mathematics, University of Technology, Sydney, Australia.* [Eckhard.Platen@uts.edu.au]

This paper derives analytic results for European style contingent claims for a stylised *minimal market model*. This model accurately reflects empirical features of modern developed markets such as leptokurtic log-returns and the 'leverage' effect. Under such a model, a change of probability measure is neither possible nor is it required, since we utilise the benchmark framework with its associated concept fair pricing. Here, the growth optimal portfolio (GOP) is used as numeraire, and hence all contingent claim prices are obtained as conditional expectations under the real world probability measure. Specifically, we calculate derivative prices for an option on a well-diversified stock index, an option on an exchange price that can in most cases be represent equity prices and in some circumstances model exchange rates, as well as options on zero coupon bonds.

# On tail distributions of supremum and quadratic variation of cádlág local martingales

Shunsuke Kaji

*Department of Mathematics, Gradurate School of Science, Osaka University*
[kaji@math.sci.osaka-u.ac.jp]

In the case of a continuous local martingale there have been works by Azema, Gundy, and Yor[1], Elworthy, Li, and Yor[2], and Takaoka[4] etc. Recently, by Liptser and Novikov[3] they were extended to the case of a local martingale with uniformly bounded jumps. We introduce the main result in that paper;

**Theorem 0.0.1** *Let $M = \{M_t\}_{t\in\mathbf{R}_+}$ be a locally square integrable cádlág martingale defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in\mathbf{R}_+}, P)$ the filtered probability space with standard general conditions. Assume that $\langle M\rangle_\infty = \lim_{t\to\infty}\langle M\rangle_t < \infty$ a.s and $\{M_\tau^+\}_{\tau\in\mathcal{T}}$ is uniformly integrable, where $\mathcal{T}$ is the set of stopping times $\tau$. Then*

(*i*) $0\leq E[M_\infty]\leq E[M_\infty^+] < \infty$.

*Besides,*

(*ii*) *if $\{\Delta M_\tau\}_{\tau\in\mathcal{T}}$ is uniformly integrable, then*

$$\lim_{\lambda\to\infty}\lambda P(\sup_{t\in\mathbf{R}_+}(M_t^-) > \lambda) = E[M_\infty];$$

(*iii*) *if $|\Delta M|\leq K$ and $E[e^{\epsilon M_\infty}] < \infty$ for some $K > 0$ and $\epsilon$, then*

$$\lim_{\lambda\to\infty}\lambda P(\sqrt{\langle M\rangle_\infty} > \lambda) = \lim_{\lambda\to\infty}\lambda P(\sqrt{[M]_\infty} > \lambda) = \sqrt{\frac{2}{\pi}}E[M_\infty].$$

In this presentation, we will present the result without the uniform boundedness assumption for jumps. But, to obtain the characterization of a tail distribution of quadratic variation of a local martingale $M$, we replace it by another assumptions: "the quasi left-continuity of $M$ and the exponential moment in terms of the compensator of the counting measure of $\Delta M$."

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