Nonreversible perturbations accelerate convergence

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To sample from distributions in high dimensional spaces or finite large sets directly is not feasible in practice, especially when the corresponding densities are known up to normalizing constants only. One has to resort to approximations. A Markov process with the underlying distribution as its equilibrium is often used to generate an approximation ("MCMC"). How good the approximation is depends on the approximating Markov process and on the specific criterion used for comparison. One may investigate the convergence properties of some particular Monte Carlo Markov processes, or compare the convergence rate within a family of Markov processes (with the same equilibrium) w.r.t. different criteria, or even try to find optimal solutions in that family. Mathematical problems arising from this approach are challenging. We prove, [1], that by simply adding a weighted divergence-free drift to a reversible diffusion, the convergence to equilibrium is accelerated. In other words, from an algorithmic point of view the nonreversible algorithm performs better. Note that different criteria are considered. The analysis is related to the study of antisymmetric perturbations of self-adjoint infinitesimal generators. Related problems will be discussed. For example the optimal solution is still open. A simulation study for two dimensional torus indicates that the rate could be infinite [2]. As for finite sample space, some preliminary results show that nonreversible perturbations accelerate convergence too.

1. Hwang, C.-R., S.-Y. Hwang-Ma, S.-J. Sheu (2005). Accelerating diffusions. To appear in *Ann. Appl. Probab.*
2. Hwang, C.-R., H.-M. Pai (2005). Optimal accelerating rate for diffusions on T^2 and S^2. In preparation.