Noncausal Problems in Stochastic Calculus

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The stochastic calculus is a calculus with respect to an underlying basic stochastic process, like the Brownian motion, say Z_t $t \in I$. It concerns the diferentiation and integration with respect to the Z_t of such random functions that appear as functionals of the $\{Z_t,\ t \in I\}$. The stochastic calculus originated by K.Itô in 1942 is founded on the fundamental *Hypothesis of Causality*, saying that; every random function $f(t, \omega)$ should be adapted to the increasing familly of σ-fields $\mathcal{F}_t$ generated by the Z_t. The hypothesis seems well fit to the principle of causality in physical sciences, where the variable "t" appears as time parameter. Moreover it endowes the theory a remarkable situation of being in natural concordance with the notion of *martingale* which plays indeed an essential role in Itô's Calculus.

Nevertheless the hypothesis of *Causality* gives a disagreable shade on the applicability of the causal theory of stochastic calculus. This can be seen immediately, for example when we think of the case that "t" stands for the *space* parameter, or in such case where the parameter "t" is multi-dimensional (that is, "a stochastic calculus" for the random field, [2]). The notion of Causality looses its sound meaning in such cases because of the lack of natural sense of *time direction*. Even in the case of physical problems where "t" appears as *time parameter*, we can find various situations of noncausal nature, such as the Cauchy problem in the theory of Brownian particle equations [3], noncausal version of the Black-Sholes model in Mathematical Finance [4],the White noise analysis [1] etc. These were the motivations for the author to introduce the noncausal theory of stochastic calculus in 1979, based on the noncausal stochastic integral which is often refered by author's name.

In this talk we will give a unified sketch of the noncausal theory of stochastic calculus as well as of its recent development. We will also refer to some typical applications of the theory to mathematical sciences.

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