

# Branching processes in random environment and the bottleneck of evolution

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Let  $Z_n$  be the number of particles at time  $n = 1, 2, \dots$ , in a branching process in random environment specified by a sequence of iid probability generating functions  $\{f_n(s)\}_{n \geq 1}$ ,  $s \in [0, 1]$ . Set  $X_k = \log f'_k(1)$ , let  $S_0 = 0, S_n = X_1 + \dots + X_n$ ,  $n \geq 1$ , be the associated random walk for this process and denote by  $\tau(n)$  the left-most point of minimum of  $S_k, k = 0, 1, \dots$ , on the interval  $0 \leq k \leq n$ . Assuming that Spitzer's condition

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n P(S_k > 0) \rightarrow \rho \in (0, 1)$$

fulfills and imposing some additional mild conditions on the characteristics of  $f_n(s)$ , we prove (under the quenched approach) conditional limit theorems for the process  $\{Z_{nt}, t \in (0, 1]\}$  conditioned on the event  $\{Z_n > 0\}$ . It is shown that up to a random multiplier, being positive and bounded with probability 1,  $Z_{nt}$  grows like  $e^{S_{nt} - S_{\tau(nt)}}$  as  $n \rightarrow \infty$  and, therefore, is subject to large oscillations. This means, in particular, that the process passes during its evolution through a sequence of "bottlenecks" which coincide with the strict descending ladder epochs of  $S_k$  within the interval  $[0, n]$ .

We also study the distribution of the distance to the closest mutual ancestor of the individuals of the  $n$ -th generation and show that the closest mutual ancestor is located in a vicinity of point  $\tau(n)$ .

**Acknowledgment.** This work was supported in part by the INTAS grant 03-51-5018, by the joint NWO-RFBR grant 047.016.013 and by the program "Contemporary problems of Fundamental Mathematics" of the Russian Academy of Sciences.