Optimal control of stochastic differential delay equations with application in economics

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This research deals with the study of optimal control of stochastic differential delay equations and their applications. By using the Dynkin formula and solution of the Dirichlet-Poisson problem, the Hamilton-Jacobi-Bellman (HJB) equation and the inverse HJB equation are derived. Application is given to a stochastic model in economics, a Ramsey model with delay and randomness .

The model is described by the equation

$$dK(t) = [BK(t-T) - u(K(t))C(t)]\, dt + \sigma(K(t-T))dw(t)$$

where K is the capital, C is the production rate, u is a control process, B is a positive constant, σ is a standard deviation of the "noise". The "initial capital"

$$K(t) = \phi(t), \quad t \in [-T, 0],$$

is a continuous bounded positive function. For this stochastic economic model the optimal control is found to be $u_{\min} = K(0) \cdot C(0)$, and the optimal performance is

$$\begin{aligned} J(K, u_{\min}) &= \tfrac{K^2(0)}{2} + \tfrac{K^2(0)\cdot C^2(0)}{2} + \textstyle\int_{-T}^{0} \phi^2(\theta)\, d\theta = \\ &= \tfrac{K^2(0)}{2}(1 + C^2(0)) + \textstyle\int_{-T}^{0} \phi^2(\theta)\, d\theta. \end{aligned}$$

The full version of this paper is submitted for publication [1]. Necessary preliminaries on stochastic differential equations and the original Ramsey model can be found in [2] and [3], respectively.

1. Ivanov, A.F. and Swishchuk, A.V. (2004). *Optimal control of stochastic differential delay equations with application in economics.* Preprint, December 2004, 12 pp. (submitted)
2. Oksendal, B. (1992). *Stochastic Differential Equations. An Introduction with Applications.* Springer-Verlag, 224 pp.
3. Ramsey, F.P. (1928). *A mathematical theory of savings.* Economic J. **38** , 543-549.