

# On stochastic processes in random environment and related topics

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It is well known that there is a correspondence between random walks in random environment in the sense of Solomon and stochastic processes in random environment in one-dimensional space. That is,

$$\text{RW in RE} \Leftrightarrow dX_t = dB_t - \frac{1}{2}\{W'(X_t) + \kappa\}dt, \quad \kappa \text{ is a constant.}$$

In particular, the case  $\kappa = 0$  (we call it a Brox diffusion, [1]) corresponds to Sinai's random walk [8] (recurrent case). We know their asymptotic behaviors are same and are of  $1/(\log x)^2$ -order. Its limit distribution is calculated by Golosov [2] and Kesten [7].

If the random environment is changed to one-sided Brownian motion, then we can obtain drastically strange phenomena [5, 6].

In high dimensional space, the papers Kalikow [4] and Zetouni [12] are well-known. In diffusion type cases, we have Tanaka's paper [10], which depends on Ichihara [3]. We try to consider the recurrence of product processes using Tomisaki's result [11].

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