Computing convergence rates for denumerable Markov chains

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Consider an irreducible, aperiodic Markov chain $\{X_n\}_{n=1,2,\ldots}$ in discrete time, on a denumerable state space $\mathcal{S}$. Suppose that there exists a contractive Lyapunov function f for this chain. That is, there exist a positive function f on $\boldsymbol{S}$, an *exception set* A, with $A, A^c := \boldsymbol{S} \setminus A \neq \emptyset$; a bounded step function $k : \boldsymbol{S} \to \mathbf{Z}_+$, i.e., $\sup_x k(x) < \infty$, and constants $\gamma, c \geq 0$, such that $\mathsf{E}_x\{f(\xi_{k(x)})\} \leq \exp\{-\gamma\} f(x)$, $x \notin A$; $\mathsf{E}_x\{f(\xi_{k(x)})\} < \infty$ for $x \in A$, and $\mathsf{E}_x\{f(\xi_1)\} \leq cf(x)$, $x \notin A$. If additionally, A is a finite set and f bounded away from 0, then that the stochastic process $r(X_n)$ converges exponentially quickly in L_1 for any function r bounded by f. The problem is how to compute explicit bounds.

In case of so-called stochastically monotone chains, Lund and Tweedie [1] have shown that the rate $\exp\{-\gamma\}$ in the Lyapunov function criterion is precisely the desired rate, provided A consists of the 'minimal' state and the step function is identically equal to 1. They use a coupling time argument.

In general, even when one can contruct a contractive Lyapunov function, it maybe be difficult to meet the conditions required by the above authors. We will discuss some methods, what to do in those cases. We will illustrate these methods with some queueing examples and a stochastically non-monotone magneto-optical trap model.

1. Lund, R.B. and Tweedie, R.L. (1996). Geometric convergence rates for stochastically ordered chains. *Math. of Operat. Res.* **21**, 182–195.